Bachelier PhD seminar

January-February 2023, IHP, Paris.

Friday 12th January

9h-9h20 **Nathan DE CARVALHO**, LPSM, Université Paris Cité and Engie Global Markets.
Title : Generalized KKT conditions for continuous time control problems with inequality constraints: application battery modelling .
Abstract : Given an exogenous price process written as the sum of a stochastic drift term and a martingale, we study an optimal trading problem with propagator model under inequality constraints on both the control (trading rate) and the integral of the control (inventory). We derive necessary and sufficient conditions by means of an infinite-dimensional generalized Lagrange multiplier rule, and characterize the unique solution to our problem in terms of Lagrange multipliers. In the case where the drift-like signal and the constraining functions are deterministic, the construction of such Lagrange coefficients is efficiently obtained as the limit of finite-dimensional fixed-point problems' solutions and is based on a novel "least-activation" principle, which constructs step-by-step the Lagrange measures associated to inventory constraints only on their activation domains by weighted combs of Diracs. We illustrate our theoretical findings on the modelling of a battery flexible storage: we formulate the problem of charging and dis-charging a battery optimally given such an asset faces physical constraints both on its (dis-)charging speed and load capacity, and solve it numerically taking into account either piece-wise linearly interpolated day-ahead prices or a sinusoidal function for the drift signal. We benchmark our novel Least-Activation numerical scheme on Uzawa's projected gradient descent and compare both algorithms. Joint work with Eduardo Abi Jaber and Huyên Pham.

9h20-9h40 **Lorenzo CROISSANT**, ENSAE
Title : Near-continuous Time Reinforcement Learning with Continuous States.
Abstract : The paradigm of Reinforcement Learning offers new perspectives for the study of Stochastic Control under uncertainty. Unfortunately, past work in the field aiming to obtain guarantees in the online non-episodic case has struggled to reach beyond simple systems (Finite States / Linear Quadratic systems). By considering event times to be random, we introduce a new framework which can employ standard methods from ergodic control. Combining these with a refined analysis of the learning problem allows us to extend existing methodologies to a rich class of non-linear control problems.

9h40-10h **Fanny CARTELLIER**, CREST, ENSAE Paris, Institut Polytechnique de Paris

Title : Can investors curb greenwashing?

Abstract : We show how investors who have pro-environmental preferences and penalize revelations of past environmental controversies impact corporate greenwashing practices. Through a dynamic equilibrium model with information asymmetry, we characterize firms' optimal environmental communication, pollution abatement, and greenwashing policies, and we explain the forces driving them. Notably, we show that, despite the threat of penalty, companies continuously greenwash as long as the gap between their environmental score and their undisclosed fundamental environmental value is not too high. We provide empirical evidence supporting the equilibrium results of our model. This is a joint work with Peter Tankov and Olivier David Zerbib.

10h-10h20 **Break**

10h20-10h40 **Azar LOUZI**, LPSM, Université Paris Cité

Title : Multilevel Stochastic Approximation of the Value-at-Risk and Expected Shortfall

Abstract : We propose a multilevel stochastic approximation (MLSA) scheme for the computation of the value-at-risk (VaR) and the expected shortfall (ES) of a financial loss, which can only be computed via simulations conditional on the realization of future risk factors. Thus, the problem of estimating its VaR and ES is nested in nature and can be viewed as an instance of a stochastic approximation problem with biased innovation. In this framework, for a prescribed accuracy ε, the optimal complexity of a standard stochastic approximation algorithm is shown to be of order ε^{-3}. To estimate the VaR, our MLSA algorithm attains an optimal complexity of order $\varepsilon^{-(2+\delta)}$, where $\delta < 1$ is some parameter depending on the integrability degree of the loss, while to estimate the ES, it achieves an optimal complexity of order $\varepsilon^{-2}|\ln \varepsilon|^2$. We also establish central limit theorems for the renormalized errors associated with both algorithms and their averaged variations. Numerical studies of the joint evolution of the error rate and the execution time demonstrate how our MLSA algorithm regains a significant amount of the lost performance due to the nested nature of the problem.

10h40-11h **Songbo WANG**, CMAP, Ecole polytechnique

Title : Uniform-in-time propagation of chaos

Abstract : In this short talk, I will introduce some recent progresses in the uniform-in-time propagation of chaos behavior of various mean field dynamics. Examples include the overdamped and kinetic mean field Langevin dynamics with convex energy, and the vortex model on the whole space with confinement potential.

Friday 26th January

9h-9h20 **Matthias RAKOTOMALALA**, CMAP Ecole Polytechnique

Title : Forward Backward Systems on Riemannian Manifolds and Controlled Dynamic Random Geometric Graphs, an Approach to Strategic Networks in Mean Field Games.

Abstract : Motivated by applications in non-cooperative large networks, we emphasize the advantages of dynamic random geometric graphs on Riemannian manifolds to represent strategic interactions in mean-field games, allowing for dynamical endogenous structures while still preserving geometrical quantities at the limit such as the coarse Ollivier curvature of the graph converging to the weighted Ricci Curvature. We then derive the equations for such games, and as we shall see, this assumption leads to the usual type of equations for Mean Field Games, except for the fact that they are here set on a manifold. Finally, we study a linear quadratic mean field game on a class of non-necessarily compact Riemannian manifolds.

9h20-9h40 **Mohamed HAMDOUCHE**, LPSM, Université Paris Cité

Title : Policy gradient learning methods for stochastic control with exit time and applications to share repurchase pricing

Abstract : We develop policy gradients methods for stochastic control with exit time in a model-free setting. We propose two types of algorithms for learning either directly the optimal policy or by learning alternately the value function (critic) and the optimal control (actor). The use of randomized policies is crucial for overcoming notably the issue related to the exit time in the gradient computation. We demonstrate the effectiveness of our approach by implementing our numerical schemes in the application to the problem of share repurchase pricing. Our results show that the proposed policy gradient methods outperform PDE or other neural networks techniques in a model-based setting. Furthermore, our algorithms are flexible enough to incorporate realistic market conditions like, e.g., price impact or transaction costs.

9h40-10h **Hervé ANDRES**, Milliman & CERMICS

Title : Path-dependent implied volatility surfaces

Abstract : We propose a new model for the coherent forecasting of both the implied volatility surfaces and the underlying asset returns. In the spirit of Guyon and Lekeufack (2023) who are interested in the dependence of volatility indices (e.g. the VIX) on the paths of the associated equity indices (e.g. the S& P 500), we first study how implied volatility can be predicted using the past trajectory of the underlying asset price. Our empirical study reveals that a large part of the movements of the at-the-money-forward implied volatility for up to two years maturities can be explained using the past returns and their squares. Moreover, we show that up to four years of the past evolution of the underlying price should be used for the prediction and that this feedback effect gets weaker when the maturity increases. Building on this new stylized fact, we fit to historical data a parsimonious version of the SSVI parameterization (Gatheral and Jacquier, 2014) of the implied volatility surface relying on only four parameters and show that the two parameters ruling the at-the-money-forward implied volatility as a function of the maturity exhibit a path-dependent behavior with respect to the underlying asset price. Finally, we propose a model for the joint dynamics of the implied volatility surface and the underlying asset price. The latter is modelled using a variant of the path-dependent volatility model of Guyon and Lekeufack and the former is obtained by adding a feedback

effect of the underlying asset price onto the two parameters ruling the at-the-money-forward implied volatility in the parsimonious SSVI parameterization and by specifying a hidden semi-Markov diffusion model for the residuals of these two parameters and the two other parameters. Thanks to this model, we are able to simulate highly realistic paths of implied volatility surfaces that are arbitrage-free.

10h-10h20 **Break**

10h20-10h40 **Anna DE CRESCENZO**, LPSM, Université Paris Cité

Title : L^2-approach to graphon mean-field systems

Abstract : We address a system of weakly interacting particles where the connections among the particles are described by a graph sequence. In the limit of the number of particles which tends to infinity, the graph sequence converges to a graphon and the system is described by a infinite collection of processes, parametrised by $[0, 1]$ and seen as a process in a suitable L^2 space, constructed via a Fubini extension. Thanks to the L^2 formulation, we are able to extend the known results in the literature and tackle interactions that depend in a non-linear way on the empirical measure. This is a joint work with Fabio Coppini and Huyên Pham.

10h40-11h **Ali BAOUAN**, CMAP Ecole polytechnique.

Title : Crediting football players for creating dangerous actions in an unbiased way: the generation of threat (GoT) indices.

Abstract : We introduce an innovative methodology to identify football players at the origin of threatening actions in a team. In our framework, a threat is defined as entering the opposing team's danger area. We investigate the timing of threat events and ball touches of players, and capture their correlation using Hawkes processes. Our model-based approach allows us to evaluate a player's ability to create danger both directly and through interactions with teammates. We define a new index, called Generation of Threat (GoT), that measures in an unbiased way the contribution of a player to threat generation. For illustration, we present a detailed analysis of Chelsea's 2016-2017 season. We are able to credit each player for his involvement in danger creation and determine the main circuits leading to threat.

Friday 2 February

9h-9h20 **Kexin SHAO**, Inria Mathrisk, Cermics Ecole des ponts; Ceremade Université Paris Dauphine.

Title : Non-decreasing martingale coupling

Abstract : The traditional optimal transport problem (OT) consists in minimizing the expected cost $\mathbb{E}[c(X_1, X_2)]$ by considering the joint distribution (μ, ν) where the marginal distributions of the random variables $X_1 \sim \mu$ and $X_2 \sim \nu$ are fixed. Motivated by financial applications,martingale optimal transport is considered adding an additional martingale constraint $\mathbb{E}[X_2|X_1] = X_1$ on top of the OT problem. Hobson and Neuberger first studied the problem with the specific cost function $c(x, y) = -|y - x|$, Juillet and Beiglbock proved the uniqueness of the associated optimizer π^{HN} when μ is continuous. We observe numerically that the π^{HN} is still a maximizer for $\rho \in (0, 2)$ and a minimizer for $\rho > 2$. We investigate the theoretical validity of this numerical observation and give rather restrictive sufficient conditions for the property to hold. We also exhibit couples (μ, ν) such that it does not hold. π^{HN} is known to satisfy some monotonicity property which we call non-decreasing. We check that the non-decreasing property is preserved for maximizers when $\rho \in (0, 1]$. In general, there exist distinct non-decreasing martingale couplings, and we find some decomposition of ν which is in one-to-one correspondence with couplings in a non-decreasing sense.

9h20-9h40 **Florin SUCIU**, CEREMADE, Université Paris Dauphine-PSL.

Title : Self-interacting approximation to McKean-Vlasov long-time limit: a Markov chain Monte Carlo method

Abstract : For a certain class of McKean-Vlasov processes, we introduce proxy processes that substitute the mean-field interaction with self-interaction, employing a weighted occupation measure. Our study encompasses two key achievements. First, we demonstrate the ergodicity of the self-interacting dynamics, under broad conditions, by applying the reflection coupling method. Second, in scenarios where the drifts are negative intrinsic gradients of convex mean-field potential functionals, we use entropy and functional inequalities to demonstrate that the stationary measures of the self-interacting processes approximate the invariant measures of the corresponding McKean-Vlasov processes. As an application, we show how to learn the optimal weights of a two-layer neural network by training a single neuron.

9h40-10h **Emmanouil SFENDOURAKIS**, CMAP, Ecole polytechnique.

Title : Understanding the worst-kept secret of high-frequency trading.

Abstract : Volume imbalance in a limit order book is often considered as a reliable indicator for predicting future price movements. In this study, we confirm this statement by analyzing an optimal control problem in which a market maker controls volumes in the limit order book of a large-tick stock and quotes prices at a half-tick distance from the mid-price. We model the mid-price, which is not a controlled variable, using uncertainty zones. The market maker has information about the underlying efficient price and consequently of the probability of a price jump in the future. By using this information , it is optimal for the market maker to create imbalances which are predictive of price movements. The value function of the market maker's control problem can be understood as a family of functions, indexed by the level of the market maker's inventory, solving a coupled system of PDEs. We show existence and uniqueness of smooth solutions for this coupled

system of equations. In the case of a continuous inventory, we also prove the uniqueness of the market maker's optimal control policy. This is joint work with Sergio Pulido and Mathieu Rosenbaum.

10h-10h20 **Break**

10h20-10h40 **Charles MEYNARD**, CMAP, Ecole polytechnique.
Title : Noise through an additional variable for mean field games master equation on finite state space
Abstract : This work consists in the mathematical study of the well-posedness of a master equation on finite state space with terms modelling common noise for which few results seems to be known at the moment. In this approach, the solution of the master equation depends on an additional variable modelling the value of a stochastic process impacting all players.

10h40-11h **Xiaoyuan (Shaun) LI**, Université Paris 1 Panthéon-Sorbonne.
Title : The quintic Ornstein-Uhlenbeck volatility model that jointly calibrates SPX & VIX smiles
Abstract : The quintic Ornstein-Uhlenbeck (OU) volatility model is a stochastic volatility model where the volatility process is a polynomial function of degree five of a single OU process with fast mean reversion and large vol-of-vol. The model can achieve remarkable joint fits of the SPX-VIX smiles with only 6 effective parameters and an input curve that allows to match certain term structures. Even better, the model remains very simple and tractable for pricing and calibration: 1) the VIX squared is again polynomial in the OU process, leading to efficient VIX derivative pricing by a simple integration with respect to the Gaussian density; 2) the model is exponential affine (in infinite dimension), leading to fast pricing of SPX derivatives via the Fourier Laplace Transform characterized by a system of infinite dimensional Riccati ODEs. Based on joint works with Eduardo Abi Jaber (Ecole Polytechnique) and Camille Illand (AXA Investment Managers).

Friday 9 February

9h-9h20 **Dorinel BASTIDE**, LaMME, Université d'Evry, Université Paris-Saclay GS Math.
Title : Stochastic modeling of financial networks dominated by clearing central counterparties and applications to bank stress test exercises
Abstract : This thesis describes a static one-period elliptical model setup for heterogeneous portfolios and joint defaults of several financial actors, also CCP members, and identifies explicit formulas for XVA costs generated by clearing and collateralized bilateral activities for handling potential losses for one of those actors. These formulas are used for exploratory regulatory stress tests and reverse stress tests. Scenarios of financial actors defaults are studied in terms of financial network re-structuration using liquidations and hedging of the defaulted positions leading to price impacts that can be resolved thanks to Radner equilibrium approach or using least cost idealized auctions for portfolio novation thanks to Markovian simulated annealing stochastic algorithms. We also analyse the monotonic property of convex risk measures applied to a financial actor loss random variable with respect to the model correlation parameters using the supermodular order properties.

9h20-9h40 **Jules DELEMOTTE**, CMAP Ecole polytechnique.
Title : Evaluating the Skew-Stickiness Ratio in stochastic and rough volatility models
Abstract : We study the dynamic properties of some classes of stochastic and rough volatility models (including well-known classical examples with their "rough volatility counterpart": the 2-factor Bergomi model, the rough Bergomi model, Heston and rough Heston). For dynamic properties, we intend the dynamics of option implied volatilities, as induced by the model. For some of the recently introduced models (notably rough volatility models), quite some effort in the literature has been concentrated on the analysis of their static properties such as their calibration power or the term structure of ATM skews but, to the best of our knowledge, their dynamic properties have received only little attention so far. One specific indicator of joint spot-price and implied volatility dynamics is the Skew-Stickiness Ratio (SSR), introduced by Bergomi [Bergomi, Smile dynamics IV, Risk 2009] and related to classical smile dynamic regimes (namely, sticky-strike and sticky-delta). We evaluate different estimators of the model SSR – mainly Monte Carlo based – and compare the results with the empirical market SSR for some large stock indices, which sheds light on the interest of using a certain modeling choice with respect to another. With a view on explicit approximation formulas, we build on the celebrated Bergomi-Guyon expansion for ATM implied volatilities and skews so to obtain explicit expansions of the model SSR, for which we analyse the accuracy with respect to our Monte Carlo benchmark. Joint work with Florian Bourgey (Bloomberg NY) and Stefano De Marco (Ecole polytechnique).

9h40-10h **Louis-Amand GERARD**, CES, Paris 1 Panthéon-Sorbonne
Title : Signature volatility models: fast pricing and hedging with Fourier
Abstract : In this talk we will present some of our work on a stochastic volatility model where the volatility is driven by a linear function of the signature of a (time extended) Brownian motion. Our main motivation is to improve the pricing and hedging method of [1, 3]. Their theory has the main advantage of being completely model-free and adapted to path dependent payoffs but at the cost of being much less tractable and to not realistically converge in practice to good approximations for non-smooth payoffs, e.g. European/Asian

calls/puts. Our aim is to show that we can improve their results by restricting to a class of Sig-SDE models (inspired by [2] but with an additional correlation between the Brownians). We do so by using Fourier techniques: provided that some infinite-dimensional Riccati equation admits a solution, we can derive the joint characteristic function of the log-price and integrated variance which allows us to price and (quadratically) hedge certain European and path-dependent options using Fourier inversion techniques. This is a joint work with Eduardo Abi Jaber.

[1] Lyons, T., Nejad, S., & Perez Arribas, I. (2020). Non-parametric pricing and hedging of exotic derivatives. Applied Mathematical Finance, 27(6), 457-494.
[2] Arribas, I. P., Salvi, C., & Szpruch, L. (2020, October). Sig-SDEs model for quantitative finance. In Proceedings of the First ACM International Conference on AI in Finance (pp. 1-8).
[3] Christa Cuchiero, Guido Gazzani, and Sara Svaluto-Ferro. Signature-based models: theory and calibration. arXiv preprint arXiv:2207.13136, 2022.

10h-10h20 **Break**

10h20-10h40 **Christian YEO**, LPSM Sorbonne Université and ENGIE Global Markets

Title : Convex ordering for stochastic control: the swing contracts case.

Abstract : We investigate propagation of convexity and convex ordering on a typical stochastic optimal control problem, namely the pricing of "*Take-or-Pay*" swing option, a financial derivative product commonly traded on energy markets. The dynamics of the underlying asset is modelled by an ARCH model with convex coefficients. We prove that the value function associated to the stochastic optimal control problem is a convex function of the underlying asset price. We also introduce a domination criterion offering insights into the monotonicity of the value function with respect to parameters of the underlying ARCH coefficients. A focus is placed on the one-dimensional setting where, by means of Stein's formula and regularization techniques, we show that the assumption of convexity for the ARCH coefficients can be relaxed with a semi-convexity assumption. To validate the results presented in this paper, we also conduct numerical illustrations.

[1] Olivier Bardou, Sandrine Bouthemy, and Gilles Pagès. When are swing options bang-bang ? International Journal of Theoretical and Applied Finance, 13(06) :867–899, September 2010.
[2] Christophe Barrera-Esteve, Florent Bergeret, Charles Dossal, Emmanuel Gobet, Asma Meziou, Rémi Munos, and Damien Reboul-Salze. Numerical methods for the pricing of swing options : A stochastic control approach. Methodology and Computing in Applied Probability, 8 :517–540, 2006.
[3] Benjamin Jourdain and Gilles Pagès. Convex ordering for stochastic Volterra equations and their euler schemes. To appear in Finance and Stochastics, 2023.
[4] Vincent Lemaire, Gilles Pagès, and Christian Yeo. Swing contract pricing : with and without neural networks. ArXiv, 2306.03822, 2023.

10h40-11h **Lionel SOPGOUI**, LPSM Université Paris Cité, Direction des risques BPCE, and Department of Mathematics Imperial College London.

Title : Impact of Climate transition on Recovery and on Loss Given Default with stochastic collaterals.

Abstract : In this work, we will revisit first the results of Bouveret and al.(2023) [1] in a continuous time setting, namely a multisectoral economic model with a productivity following a multidimensional Ornstein-Uhlenbeck and continuous carbon tax, a firm valuation model, and credit risk model. Secondly, we will define

the loss at default as the difference between Exposure at Default (EAD) and the liquidated collateral, which will help us to define model the Loss Given Default (LGD) - the expected percentage of exposure that is lost if a debtor defaults. If the collateral is a financial asset, we will model it in a third step by the continuous time version of the discounted cash flows, where the cash flows SDE is driven by the instantaneous consumption growth, the instantaneous growth of a tax's function and a Brownian motion. Finally, if the collateral is a property in housing market, we will model it by an exponential Ornstein- Uhlenbeck to which we add a jump Poisson process describing the transition scenario such as energy efficiency. We obtain thus how the loss' risk measures of a credit portfolio are skewed in the context of climate transition through carbon taxes and/or energy performance of buildings.

[1] G. Bouveret, J.-F. Chassagneux, S. Ibbou, A. Jacquier, and L. Sopgoui, Propagation of carbon tax in credit portfolio through macroeconomic factors, arXiv preprint arXiv:2307.12695, (2023).

Friday 7th June

9h-9h20 **Edoardo LOMBARDO**,
Title : TBA.
Abstract : TBA.

9h20-9h40 **Redouane SILVENTE**, ENSAE Paris, CREST
Title : TBA
Abstract : TBA.

9h40-10h **Rengim CETINGOZ**,
Title : TBA
Abstract : TBA.

10h-10h20 **Break**

10h20-10h40 **Nisrine MADHAR**, LPSM, Université Paris Cité.
Title : Tail-Related Risk Measures Estimation using Stochastic Simulation of Extremes
Abstract : In the context of financial market stress testing, in which the focus is more on financial crisis, the best tailored tools to capture the risk that such events could represent for financial institutions are tail-related risk measures. Hence, extraordinary market circumstances are of upmost importance, and they translate through the estimation of risk measures at extreme levels. However, the data size in these regions can be rather small making the estimation a challenging task. In addition, the computation of these risk measures usually only relies on the risk factor of interest. However, it could be shown that these univariate assessments could be improved if one is able to identify a set of risk factors exhibiting asymptotic dependence with the risk factor of interest, and then include them in the quantification of the risk of the risk factor of interest. The generalization of the simulation approach suggested in [1] to a dimension $K > 2$ solves these issues. It aims at expanding jointly and conditionally a sample of extremes, for a set of asymptotically dependent risk drivers through stochastic simulation based on a specific characterization of multivariate extremes with Multivariate Generalized Pareto Distribution (MGPD) see [2]. Numerical study shows the true benefit of employing this approach in terms of the estimates accuracy, over empirical and sophisticated existing approaches. This is a joint work with Maud Thomas.
[1] Juliette Legrand, Pierre Ailliot, Philippe Naveau, and Nicolas Raillard. Joint stochastic simulation of extreme coastal and offshore significant wave heights. 2023.
[2] Holger Rootzen, Johan Segers, and Jennifer L Wadsworth. Multivariate generalized pareto distributions: Parametrizations, representations, and properties. Journal of Multivariate Analysis, 165: 117–131, 2018.

10h40-11h **Nathan SAULDUBOIS**, CMAP Ecole polytechnique.
Title : TBA.

Abstract : TBA.